GCE AS/A level

0975/01

MATHEMATICS – C3
Pure Mathematics

A.M. WEDNESDAY, 23 January 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_1^2 \frac{1}{2+e^x}\,dx.$$

Show your working and give your answer correct to three decimal places. [4]

2. *(a)* (i) Show, by counter-example, that the statement

$$\cos^3\theta \equiv 1 - \sin^3\theta$$

is false.

(ii) **Write down** a value of θ which does satisfy the equation

$$\cos^3\theta = 1 - \sin^3\theta.$$ [3]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$4\operatorname{cosec}^2\theta = 9 - 8\cot\theta.$$ [6]

3. *(a)* Given that

$$x^3 + 5x^4y - 2y^3 + 7 = 0,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

(b) Given that $x = t^3 - 5$, $y = t^4 + 7t^5$,

(i) find an expression for $\frac{dy}{dx}$ in terms of t,

(ii) find an expression for $\frac{d^2y}{dx^2}$ in terms of t,

(iii) find the value of $\frac{d^2y}{dx^2}$ when $x = 3$. [9]

4. *(a)* On the same diagram, sketch the graphs of $y = \ln x$ and $y = 11 - 2x$.
Deduce the number of roots of the equation

$$\ln x + 2x - 11 = 0.$$ [3]

(b) **You may assume** that the equation

$$\ln x + 2x - 11 = 0$$

has a root α between 4 and 5.
The recurrence relation

$$x_{n+1} = \frac{11 - \ln x_n}{2},$$

with $x_0 = 4{\cdot}7$, can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [5]

5. *(a)* Differentiate each of the following with respect to x.

(i) $\sqrt{5x^2 - 3x}$ (ii) $\sin^{-1} 7x$ (iii) $e^{3x} \ln x$ [7]

(b) By first writing $\cot x = \dfrac{\cos x}{\sin x}$, show that $\dfrac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$. [3]

6. *(a)* Find

(i) $\displaystyle\int \cos\left(\frac{4x+5}{3}\right) dx$, (ii) $\displaystyle\int e^{2x+9}\, dx$, (iii) $\displaystyle\int \frac{3}{(7-2x)^6}\, dx$. [6]

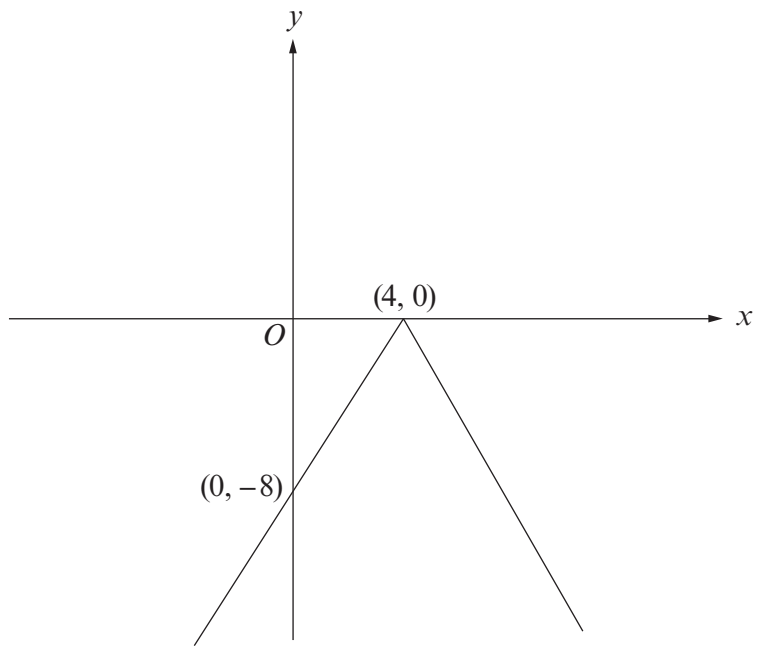
(b) Express $\displaystyle\int_2^{44} \frac{1}{3x-4}\, dx$

in the form $\ln k$, where k is an integer whose value is to be found. [4]

7. *(a)* Solve the inequality $|3x - 4| > 5$. [3]

(b) (i) Sketch the graph of $y = |x|$.

(ii) The diagram below shows a sketch of the graph of $y = a|x + b|$, where a and b are constants. The graph meets the x-axis at the point (4, 0) and the y-axis at the point (0, –8).



Find the value of a and the value of b. [3]

TURN OVER

8. The function f has domain $[-1, \infty)$ and is defined by

$$f(x) = \ln(4x + 5) - 2.$$

(a) Find an expression for $f^{-1}(x)$. [4]

(b) State the domain of f^{-1}. [1]

9. *(a)* The functions f and g have domains $(-\infty, \infty)$ and $(0, \infty)$ respectively and are defined by

$$f(x) = x^2 - 25,$$
$$g(x) = 2x - 3.$$

(i) Write down the domain of fg.

(ii) Write down the range of fg.

(iii) Write down an expression for $fg(x)$.

(iv) Solve the equation $fg(x) = 0$. [7]

(b) The function h is defined by

$$h(x) = \frac{2x + 7}{5x - 2}.$$

(i) Show that $hh(x) = x$.

(ii) **Hence** write down an expression for $h^{-1}(x)$. [3]